SOLUTIONS TO IIT-JEE 2010
MATHEMATICS: Paper-I (Code: 08)

PART – II

SECTION – I

Single Correct Choice Type

This section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Note: Questions with (*) mark are from syllabus of class XI.

29. The value of $\lim_{x\to 0}\frac{1}{x^3}\int_0^x \frac{t\ln(1+t)}{t^4+4}dt$ is

(A) 0 (B) $\frac{1}{12}$ (C) $\frac{1}{24}$ (D) $\frac{1}{64}$

Sol.: **Correct choice: (B)**

30. The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $\boldsymbol{A}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$ has exactly two distinct solutions, is

(A) 0 (B) 2^9-1 (C) 168 (D) 2

Sol.: **Correct choice: (A)**

31. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}$, $4\hat{i}$, $3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral $PQRS$ must be a

(A) parallelogram, which is neither a rhombus nor a rectangle (B) square
(C) rectangle, but not a square (D) rhombus, but not a square

Sol.: **Correct choice: (A)**

32. Let ω be a complex cube root of unity with $\omega\neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1}+\omega^{r_2}+\omega^{r_3}=0$ is

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$ (C) $\frac{2}{9}$ (D) $\frac{1}{36}$

Sol.: **Correct choice: (C)**

33. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

(A) $x+2y-2z=0$ (B) $3x+2y-2z=0$ (C) $x-2y+z=0$ (D) $5x+2y-4z=0$

Sol.: **Correct choice: (C)**

***34.** If the angles A, B and C of a triangle are in an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A$ is

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) 1 (D) $\sqrt{3}$

Sol.: **Correct choice: (D)**

35. Let f, g and h be real-valued functions defined on the interval [0, 1] by $f(x)=e^{x^2}+e^{-x^2}$, $g(x)=xe^{x^2}+e^{-x^2}$ and $h(x)=x^2e^{x^2}+e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f, g and h on [0, 1], then

(A) $a=b$ and $c\neq b$ (B) $a=c$ and $a\neq b$ (C) $a\neq b$ and $c\neq b$ (D) $a=b=c$

Sol.: Correct choice: (D)

***36.** Let p and q be real numbers such that $p\neq 0,\ p^3\neq q$ and $p^3\neq -q$. If α and β are nonzero complex numbers satisfying $\alpha+\beta=-p$ and $\alpha^3+\beta^3=q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

(A) $(p^3+q)x^2-(p^3+2q)x+(p^3+q)=0$ (B) $(p^3+q)x^2-(p^3-2q)x+(p^3+q)=0$

(C) $(p^3-q)x^2-(5p^3-2q)x+(p^3-q)=0$ (D) $(p^3-q)x^2-(5p^3+2q)x+(p^3-q)=0$

Sol.: Correct choice: (B)

SECTION – II
Multiple Correct Choice Type

This section contains **5 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

***37.** Let ABC be a triangle such that $\angle ACB=\frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of x for which $a=x^2+x+1$, $b=x^2-1$ and $c=2x+1$ is (are)

(A) $-\left(2+\sqrt{3}\right)$ (B) $1+\sqrt{3}$ (C) $2+\sqrt{3}$ (D) $4\sqrt{3}$

Sol.: Correct choice: (B)

38. Let f be a real-valued function defined on the interval $(0,\infty)$ by $f(x)=\ln x+\int_0^x\sqrt{1+\sin t}\,dt$. Then which of the following statement(s) is (are) true?

(A) $f''(x)$ exists for all $x\in(0,\infty)$

(B) $f'(x)$ exists for all $x\in(0,\infty)$ and f' is continuous on $(0,\infty)$, but not differentiable on $(0,\infty)$

(C) there exists $\alpha>1$ such that $|f'(x)|<|f(x)|$ for all $x\in(\alpha,\infty)$

(D) there exists $\beta>0$ such that $|f(x)|+|f'(x)|\leq\beta$ for all $x\in(0,\infty)$

Sol.: Correct choice: (B), (C)

***39.** Let A and B be two distinct points on the parabola $y^2=4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be

(A) $-\frac{1}{r}$ (B) $\frac{1}{r}$ (C) $\frac{2}{r}$ (D) $-\frac{2}{r}$

Sol.: Correct choice: (C), (D)

40. The value(s) of $\int_0^1\frac{x^4(1-x)^4}{1+x^2}dx$ is (are)

(A) $\frac{22}{7}-\pi$ (B) $\frac{2}{105}$ (C) 0 (D) $\frac{71}{15}-\frac{3\pi}{2}$

Sol.: Correct choice: (A)

***41.** Let z_1 and z_2 be two distinct complex numbers and let $z = (1-t)z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg(w) denotes the principal argument of a nonzero complex number w, then

(A) $|z - z_1| + |z - z_2| = |z_1 - z_2|$

(B) $\text{Arg}(z - z_1) = \text{Arg}(z - z_2)$

(C) $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

(D) $\text{Arg}(z - z_1) = \text{Arg}(z_2 - z_1)$

Sol.: Correct choice: (A), (C), (D)

SECTION – III
Comprehension Type

This section contains **2 paragraphs**. Based upon the first paragraph **3 multiple choice questions** and based upon the second paragraph **2 multiple choice questions** have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions 42 to 44

Let p be an odd prime number and T_p be the following set of 2 × 2 matrices :

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \ldots\ldots, p-1\} \right\}$$

42. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det (A) divisible by p is

(A) $(p-1)^2$ (B) $2(p-1)$ (C) $(p-1)^2 + 1$ (D) $2p - 1$

Sol.: Correct choice: (D)

43. The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is

[**Note :** The trace of a matrix is the sum of its diagonal entries.]

(A) $(p-1)(p^2 - p + 1)$ (B) $p^3 - (p-1)^2$ (C) $(p-1)^2$ (D) $(p-1)(p^2 - 2)$

Sol.: Correct choice: (C)

44. The number of A in T_p such that det (A) is not divisible by p is

(A) $2p^2$ (B) $p^3 - 5p$ (C) $p^3 - 3p$ (D) $p^3 - p^2$

Sol.: Correct choice: (D)

***Paragraph for Questions 45 to 46**

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B.

45. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(A) $2x - \sqrt{5}y - 20 = 0$ (B) $2x - \sqrt{5}y + 4 = 0$ (C) $3x - 4y + 8 = 0$ (D) $4x - 3y + 4 = 0$

Sol.: Correct choice: (B)

46. Equation of the circle with AB as its diameter is

(A) $x^2 + y^2 - 12x + 24 = 0$

(B) $x^2 + y^2 + 12x + 24 = 0$

(C) $x^2 + y^2 + 24x - 12 = 0$

(D) $x^2 + y^2 - 24x - 12 = 0$

Sol.: Correct choice: (A)

SECTION – IV
Integer Answer Type

This section contains **TEN questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The correct digit below the question numbers in the ORS is to be bubbled.

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47. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of $\left(2\vec{a}+\vec{b}\right)\cdot\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}-2\vec{b}\right)\right]$ is

Sol.: **Ans. 5**

***48.** The line $2x + y = 1$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

Sol.: **Ans. 2**

49. If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then $|d|$ is

Sol.: **Ans. 6**

50. For any real number *x*, let [*x*] denote the largest integer less than or equal to *x*. Let *f* be a real valued function defined on the interval [–10, 10] by $f(x) = \begin{cases} x - [x] & \text{if } [x] \text{ is odd,} \\ 1 + [x] - x & \text{if } [x] \text{ is even} \end{cases}$. Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x \, dx$ is

Sol.: **Ans. 4**

51. Let ω be the complex number $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$. Then the number of distinct complex numbers *z* satisfying $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ is equal to

Sol.: **Ans. 1**

***52.** Let S_k, $k = 1, 2, ..., 100$, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!} + \sum_{k=1}^{100}\left|\left(k^2 - 3k + 1\right)S_k\right|$ is

Sol.: **Ans. 3**

53. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations $(y+z)\cos 3\theta = (xyz)\sin 3\theta$; $x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$; $(xyz)\sin 3\theta = (y + 2z)\cos 3\theta + y\sin 3\theta$ have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is

Sol.: **Ans. 3**

54. Let *f* be a real-valued differentiable function on **R** (the set of all real numbers) such that $f(1) = 1$. If the y-intercept of the tangent at any point $P(x, y)$ on the curve $y = f(x)$ is equal to the cube of the abscissa of *P*, then the value of $f(-3)$ is equal to

Sol.: **Ans. 9**

***55.** The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is

Sol.: **Ans. 3**

***56.** The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

Sol.: **Ans. 2**